

Dr. Campbell's Memoranda of 1907 and 1912

INTRODUCTORY NOTE

AS mentioned in the preceding article by Dr. Jewett, the first and second of the three following memoranda were the basis of methods of designing transpositions for voice-frequency circuits. They applied particularly to non-loaded circuits but the theory was readily extended to cover loaded circuits. In an earlier and more general study written in 1904, Dr. Campbell considered the involved equations necessary to an exact solution of the crosstalk problem and deduced simplifying approximations and convenient artifices for avoiding lengthy derivations.

He first assumes a line having the circuits substantially perfectly balanced to each other by means of very frequent transpositions. He then considers the effect of an unbalanced condition in a short length of line such as might arise from an irregularity in wire or transposition spacing or an unbalanced series impedance which might be due to a poor joint. Dr. Campbell refers to such effects as "slight alterations in the impedances, mutual impedances and admittances of the system." He shows how the crosstalk can be readily computed if these alterations in impedance and admittance are known.

He then considers the case of a short untransposed length in which the coupling between circuits is systematic rather than accidental. He shows that the crosstalk in such a short length can be computed in terms of mutual impedances and admittances in just the same manner used for accidental coupling in a short length nominally perfectly balanced. He shows that the mutual impedance per unit length (which is substantially proportional to the mutual inductance) is a measure of the crosstalk effect of the magnetic field of the disturbing circuit and can be computed from a knowledge of the spacing and diameters of the wires of the disturbing and disturbed circuits. The mutual admittance per unit length is a measure of the crosstalk effect of the electric field of the disturbing circuit and is shown to be proportional to the "direct capacity unbalance" which may readily be measured or computed from measurements of the individual direct capacities. This notion of direct capacity unbalance which was deduced in the earlier memorandum of 1904 has been of the greatest usefulness in crosstalk problems both in open wire and in cable. The mutual admittance defined in this way takes account not only of the

charges on the wires of the disturbing circuit but also of the charges induced by the disturbing circuit on all other wires. Thus, the electric shielding effect of other wires is taken into account. Magnetic shielding is ignored since this is unimportant with a line transposed at intervals very small compared with the wave-length. Dr. Campbell gave data for comparing the relative importance of the electric and magnetic components of the disturbing field and showed that for severe exposures both effects are of importance.

The equations given in the latter part of the memorandum of September 14, 1907, formed a basis of transposition design. They show how the crosstalk in a long transposed line may be computed with sufficient accuracy by simply summing up the effects computed individually for each short element of line. This important approximation is discussed in some detail in the earlier memorandum of 1904. Dr. Campbell prophetically says, "It must, however, always be borne in mind that we are working only with a first approximation and that in certain cases it may be necessary to continue the investigation to a higher order of approximation." In making the approximation, Dr. Campbell was, of course, thinking of voice frequency telephone circuits and at such frequencies, if the interval between transpositions is sufficiently short to guard against noise due to irregular power exposures, that interval will be but a very small fraction of the wave-length and it is unnecessary to consider the second approximation or as Dr. Campbell says to calculate "crosstalk-of-crosstalk."

When transpositions were designed for carrier frequency operation up to 30 kc. it was obviously impracticable to make a transposition interval a very small fraction of the wave-length and "crosstalk-of-crosstalk" could not be ignored. In other words, it was necessary to consider the crosstalk in each short element of line from the disturbing circuit into all the other wires on the line, the propagation of these crosstalk currents (and charges) along the line, and their effect in inducing currents in the disturbed circuit in other short elements of line. This effect has been termed interaction crosstalk since it takes account of the interaction between elements of line instead of simply summing up individual effects in each element. Thus it indeed proved true that "in certain cases" it was necessary to continue the investigation to a higher order of approximation.

I. CROSSTALK FORMULÆ FOR NON-LOADED CIRCUITS *

Take first the simple case of two perfectly symmetrical uniform circuits having the same transmission constants, which terminate at

* Memorandum dated September 14, 1907.

the same places in sets having the same impedance as the lines. Transmission upon either circuit can then give rise to no crosstalk upon the other circuit. Circuits such as two well transposed pairs on a pole lead are here to be understood; that is, there may be any number of circuits in the system but the mutual impedances and admittances between conductors connect points which are equi-spaced with respect to the impedances in each conductor.

Now suppose that at a point distanced x from the transmitting end of the circuits, slight alterations are made in the impedances, mutual impedances and admittances of the system. The effect of each change will be small and the total effect will be approximately equal to the sum of the individual results. That is, we may neglect the second-order terms, or crosstalk of crosstalk. Furthermore, unbalancing one of the given circuits alone cannot produce crosstalk. It is necessary that both circuits be unbalanced simultaneously by a single change in the system. Now, adding impedances to either side of one of the given circuits or to any third circuit will not unbalance both of the original circuits. Mutual impedance or admittance between the two sides of any circuit does not unbalance the circuit. Mutual impedance or admittance, added between either of the given circuits and any third circuit of the system, will not unbalance both of the given circuits. This leaves admittance shunted directly from one given circuit to the other given circuit and mutual impedance between the two circuits as the only source of crosstalk.

Let the admittances added between the two circuits be a, b, c, d connected between conductors 1 and 3, 3 and 2, 2 and 4, 4 and 1, respectively, where conductors 1/2 form one circuit and conductors 3/4 form the other circuit. These admittances may be resolved into the sum and difference of four admittances, as shown by the following table:

1 to 3	$a =$	$\frac{(a+b+c+d)}{4}$	$+$	$\frac{(a+b-c-d)}{4}$	$+$	$\frac{(a-b-c+d)}{4}$	$+$	$\frac{(a-b+c-d)}{4}$
3 to 2	$b =$		$+$		$-$		$-$	
2 to 4	$c =$		$-$		$-$		$+$	
4 to 1	$d =$		$-$		$+$		$-$	

By the principle of superposition the effect of the given admittances a, b, c, d will be practically the same as the sum of the effects of the four component admittances taken individually. The first component admittance $(a + b + c + d)/4$ is added symmetrically between the two wires of one circuit and the two wires of the other circuit. This will not disturb the symmetry of either circuit and will, consequently, not

give rise to crosstalk. The second admittance is added between the conductor 3 and conductors 1/2 and subtracted between conductor 4 and conductors 1/2. This does not destroy the symmetry of circuit 1/2, and it can in consequence not give rise to crosstalk. The third admittance $(a - b - c + d)/4$ is added in the same way as the second with an interchange of circuits. It will also not give rise to crosstalk.

Crosstalk due to the added admittances, a, b, c, d , must therefore be due to the last component $(a - b + c - d)/4 = Y/4$ where Y is what we may call the direct admittance unbalance.

In order to determine the crosstalk occasioned by this admittance unbalance Y , when the electromotive force E is impressed upon one of the circuits, we may proceed as follows:

The circuits are connected as shown by Fig. 1. This is equivalent

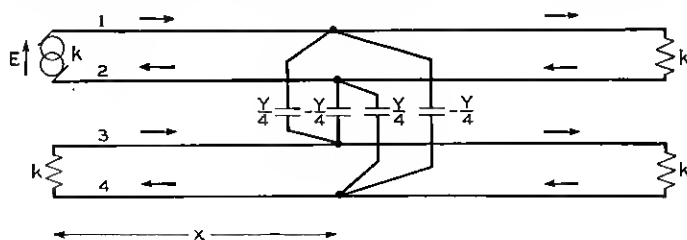


Fig. 1.

to the bridge of Fig. 2. For if the unbalancing admittances $Y/4$ were removed circuit 1/2 would be clear. Then as the e.m.f. E acts through an impedance k upon a line whose impedance is k , the potential difference at the sending end of the line would be $E/2$ and in traversing a distance x this would be attenuated by the factor $e^{-\gamma x}$. The im-

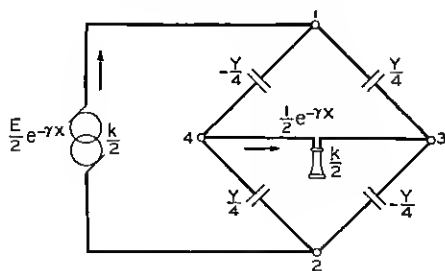


Fig. 2.

pedance of each end of circuit 1/2 at the point x will be k and therefore the entire circuit with its two ends in parallel will have the impedance $k/2$. We may therefore replace circuit 1/2 of Fig. 1 by a branch in

Fig. 2 having the impedance $k/2$ and containing an e.m.f. $Ee^{-\gamma z}/2$. Similarly circuit 3/4 will be replaced in Fig. 2 by a branch having the impedance $k/2$. As the current reaching circuit 3/4 will divide equally between the two ends of the line and the part reaching the beginning of the line will be further attenuated by the factor $e^{-\gamma z}$ the termination k in Fig. 1 is to be replaced by the receiver in Fig. 2 which indicates only $e^{-\gamma z}/2$ of the current flowing through it. Substituting these values in the expression for the galvanometer current in a bridge¹ we find for the crosstalk current

$$\Delta I_v = -\frac{EY}{16}e^{-2\gamma z} \left[\frac{1}{1 - \frac{k^2 Y^2}{64}} \right]$$

or approximately

$$\Delta I_v = -\frac{EY}{16}e^{-2\gamma z} \text{ as } Y \text{ is very small.}$$

This is the current at the end corresponding to the transmitting station. At the other end the attenuation factor will be that corresponding to transmission over the entire length of line l —making the crosstalk

$$\Delta I_v' = \frac{EY}{16}e^{-\gamma l}.$$

It will be noticed that the crosstalk at the farther end of the line is independent of the position of the admittance unbalance, while the crosstalk at the transmitting end of the system will diminish as the point of unbalance is moved farther from this end. In one case the wave must traverse the entire distance between terminal stations; in the other it must travel down the line to the point where it is carried across from one circuit to the other and then back from this point to the beginning of the line where the crosstalk is received.

The mutual impedances between the four conductors composing the two circuits may be treated in a manner similar to that which has been employed for the admittances between these conductors. Assume that any four mutual impedances are added and then divide them into four components, of which three may be shown to give rise to no crosstalk. The remaining component is the mutual impedance unbalance $Z/4$ and the crosstalk due to it may be found as follows. The circuit is shown by Fig. 3, which may be replaced by the trans-

¹ Maxwell I, § 347.

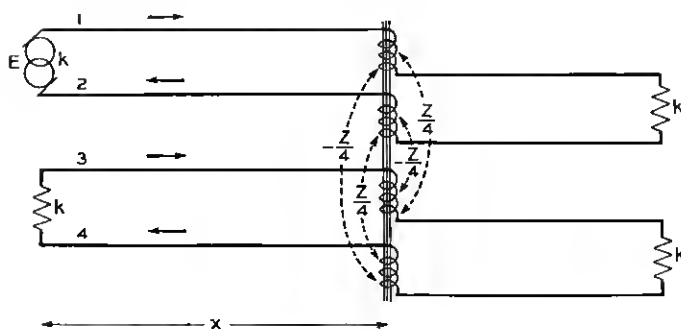


Fig. 3.

former circuit of Fig. 4. If there is no unbalance Z so that we have the original uniform system the current $E/2k$ starts out on 1/2 and at the point of mutual impedance unbalance it becomes $Ee^{-\gamma x}/2k$. Circuit 1/2 therefore behaves like a primary of impedance $2k$ (for the two ends of the line are in series) containing e.m.f. $Ee^{-\gamma x}$. The circuit

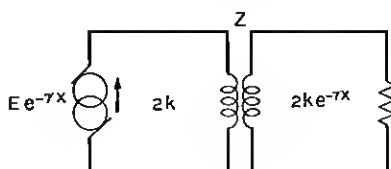


Fig. 4.

3/4 acts as a secondary of impedance $2k$. If the mutual impedance between the two lines is now made Z without any other change the current on 3/4 at x is

$$-\frac{EZe^{-\gamma x}}{4k^2 - Z^2},$$

which is attenuated by the factor $e^{-\gamma x}$ in reaching the transmitting end of the line, making the crosstalk

$$\Delta I_z = -\frac{EZ}{4k^2} e^{-2\gamma x} \left[\frac{1}{1 - \frac{Z^2}{4k^2}} \right]$$

or approximately

$$\Delta I_z = -\frac{EZ}{4k^2} e^{-2\gamma x} \text{ as } Z \text{ is small.}$$

At the farther end of the listening circuit, the approximate expression is

$$\Delta I_z' = -\frac{EZ}{4k^2}e^{-\gamma l}.$$

The total crosstalk due to the admittance and mutual impedance unbalance Y and Z is thus:

$$\Delta I = -E \left[\frac{Y}{16} + \frac{Z}{4k^2} \right] e^{-2\gamma x} \text{ at the transmitting end,}$$

$$\Delta I' = +E \left[\frac{Y}{16} - \frac{Z}{4k^2} \right] e^{-\gamma l} \text{ at the receiving end,}$$

from which we see that the ratio of the crosstalk due to the two kinds of unbalance is independent of the location of the point on the circuit at which these unbalances are introduced, and the two will be numerically equal in case:

$$Y = \frac{4}{k^2}Z$$

or

$$Y = \frac{4}{(667)^2}Z = 9 \cdot 10^{-6}Z, \text{ on non-loaded open wires.}$$

Appended tables give the direct capacity unbalances and the mutual impedance unbalances for a 40-wire lead of No. 12 wire. The relative importance of the two in producing crosstalk is shown below for a few typical cases:

	Unbalance per Mile		Ratio $\frac{9 \cdot 10^{-6}Z}{Y}$
	Direct Capacity Unbalance mmf.	Mutual Inductance Unbalance mh.	
Pairs adjacent on same cross-arm			
1/2 and 3/4.....	-1014	- 93	0.8
3/4 and 5/6.....	-1469	-143	0.9
Pairs adjacent in vertical plane			
1/2 and 11/12.....	602	72	1.1
3/4 and 13/14.....	409	72	1.6
5/6 and 15/16.....	704	147	1.9
Pairs separated by two steps			
1/2 and 5/6.....	- 116	- 31	2.4
3/4 and 7/8.....	- 84	- 19	-2.0
1/2 and 13/14.....	- 93	2	-0.2
3/4 and 15/16.....	- 150	4	-0.2
1/2 and 21/22.....	136	20	1.3
5/6 and 25/26.....	46	43	8.4

It will be noted that the two are of about equal importance for the cases of most severe static exposure. For 5/6 and 15/16, which is quite a severe exposure, the magnetic is twice as important as the static. For the pairs which are so far removed as to bring in considerable static shielding from the other wires the magnetic crosstalk may be still more important relatively. Thus for 5/6 and 25/26 it is eight times as great as the static.

At the sending end of the system the static and the magnetic cross-talks combine, while at the other end they tend to cancel each other. In practice, therefore, the summation is the more important case.

If both unbalances are pure reactances, the one being pure capacity and the other pure inductance, and the line has approximately the same impedance at all frequencies, the character of the crosstalk will be the same whether it is produced by capacity or mutual inductance. This will be approximately the case on well insulated open-wire lines. On non-loaded cable circuits the line impedance decreases as the frequency rises. On cables, therefore, the crosstalk due to mutual impedance will have the higher frequencies more strongly pronounced than the crosstalk due to capacity.

For a transposed line we find the total crosstalk by integrating the crosstalk throughout the entire length of the line. We will assume that the lines are infinitely long and that the transpositions give the system a periodic structure of lengths. Let x be the distance to the first transposition, $a, b, c, d \dots s$, the distances from the transposition to the others in the periodic section.

$$\begin{aligned} I &= -E \left(\frac{Y}{16} + \frac{Z}{4k^2} \right) \int_{0, x+a, x+a, \dots}^{x, x, x+b, x+b, \dots} e^{-2\gamma x} dx \\ &= -\frac{E}{2\gamma} \left(\frac{Y}{16} + \frac{Z}{4k^2} \right) (1 - 2e^{-2\gamma x} + 2e^{-2\gamma(x+a)} \\ &\quad \dots - 2e^{-2\gamma(x+s)} + \dots), \end{aligned}$$

since the periodic section s must contain an even number of transpositions.

$$I = -\frac{E}{2\gamma} \left(\frac{Y}{16} + \frac{Z}{4k^2} \right) \left(1 - 2e^{-2\gamma x} \frac{1 - e^{-2\gamma a} + e^{-2\gamma b} - \dots}{1 - e^{-2\gamma s}} \right)$$

or approximately

$$I = -\frac{E}{2\gamma} \left(\frac{Y}{16} + \frac{Z}{4k^2} \right) \left\{ 1 - 2[1 - 2\gamma x + 2\gamma^2 x^2] \right\}$$

$$\times \frac{2\gamma(a-b+c\cdots) - 2\gamma^2(a^2-b^2+\cdots) + \frac{4\gamma^3}{3}(a^3-b^3+\cdots)}{2\gamma s - 2\gamma^2 s^2 + \frac{4}{3}\gamma^3 s^3} \left\{ \right.$$

But the sum of alternate intervals in the transposition periodic section s must equal $s/2$. Therefore $a-b+c\cdots = a+(c-b)+(e-d)+\cdots = s/2$, and to the same approximation,

$$\begin{aligned} I &= -\frac{E}{2\gamma} \left(\frac{Y}{16} + \frac{Z}{4k^2} \right) \left\{ 1 - [1 - 2\gamma x + 2\gamma^2 x^2] \right. \\ &\quad \times \frac{1 - \frac{2\gamma}{s}(a^2-b^2+\cdots) + \frac{4\gamma^2}{3s}(a^3-b^3+\cdots)}{1 - \gamma s + 2\gamma^2 s^2/3 - \cdots} \left. \right\} \\ &= \frac{E}{2} \left(\frac{Y}{16} + \frac{Z}{4k^2} \right) s \left[\left\{ 1 - \frac{2x}{s} - \frac{2(a^2-b^2+\cdots)}{s^2} \right\} \right. \\ &\quad + \left\{ \frac{1}{3} - \frac{2x}{s} + \frac{2x^2}{s^2} - 2 \left(1 - \frac{2x}{s} \right) \frac{a^2-b^2+\cdots}{s^2} \right. \\ &\quad \left. \left. + \frac{4}{3s^3}(a^3-b^3+\cdots) \right\} \gamma s \right]. \end{aligned}$$

It will be noticed that the crosstalk varies linearly with the distance from the first transposition, approximately, and that by a suitable choice for this distance the crosstalk may be reduced to zero to the first approximation. This is, however, not a matter of especial practical importance, for incidental irregularities contribute to the crosstalk in practice. As soon as the crosstalk due to the regular transposition system is reduced to the order of that due to the accidental irregularities further reduction of this crosstalk is not a matter of commercial importance. The accidental irregularities in the distribution of the wires therefore set a limit to the extent to which it is worth while to reduce the length of the transposition sections.

If there are but two transpositions in the periodic interval $a = s/2$, $b = c = \cdots = 0$

$$I = \frac{E}{2} \left(\frac{Y}{16} + \frac{Z}{4k^2} \right) s \left[\left(\frac{1}{2} - \frac{2x}{s} \right) - \frac{x}{s} \left(1 - \frac{2x}{s} \right) \gamma s \right] \text{ approximately.}$$

If $x = s/4$, I vanishes as to first order terms. If $x = s/2$ or 0 , I has its maximum value,

$$\frac{E}{2} \left(\frac{Y}{16} + \frac{Z}{4k^2} \right) \frac{s}{2}.$$

II. CROSSTALK FORMULÆ FOR PHANTOM CIRCUITS *

There may be crosstalk between two phantom circuits, between a phantom and a distinct two-wire circuit, or between a phantom and one of its own side circuits.

In the first case we may assume that each side of both phantoms is perfectly balanced with respect to every part of the system, as we are not here concerned with the crosstalk on the side circuits. The two wires forming the side of a phantom may then be treated as a single conductor. The sources of crosstalk will therefore be direct admittance unbalance and mutual impedance unbalance exactly as for ordinary pairs. The capacity unbalance in formulæ¹ is found from the 16 direct capacities between the two sets of four wires, and the mutual impedance unbalance is found from the 16 mutual impedances between the same wires, but this is to be divided by four in order to allow for the division of the current between the two wires on each side of both phantom circuits.

In the second case each side of the phantom circuit may be assumed perfectly balanced. In the computation of the capacity unbalance, 8 direct capacities enter. There are also 8 mutual impedances involved in the mutual impedance unbalance, and these must be divided by 2 in order to allow for the division of the phantom circuit current between the two wires.

The crosstalk between a phantom circuit and one of its side circuits differs materially from the others, as the use of the same conductors to form the side circuit and one side of the phantom circuit introduces two additional sources of unbalance. These are: unbalance in the impedance of the two conductors forming the side circuit, and unbalance in the direct admittance from the two conductors forming the side circuit to the system outside of the phantom conductors.

The assumed distribution of unbalances is shown by Fig. 1. Insert

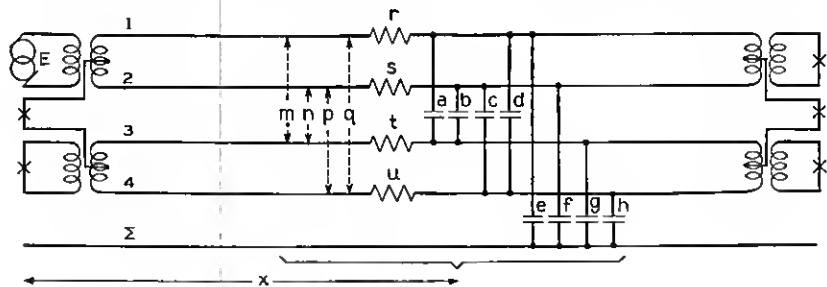


Fig. 1.

* Memorandum dated October 31, 1907.

¹ See September 14 memorandum.

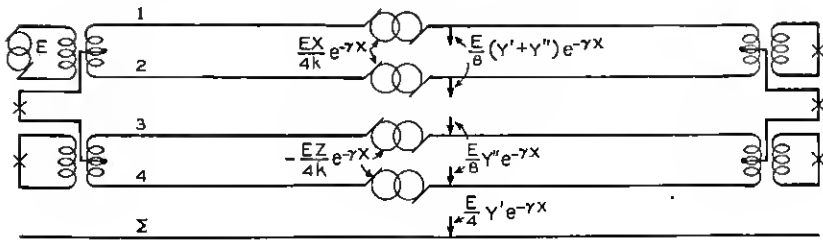


Fig. 3.

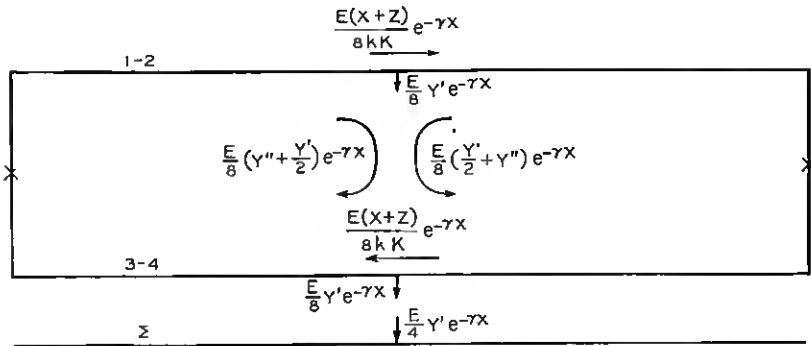


Fig. 4.

whence the current produced is $Ee^{-\gamma x}(X + Z)/8kK$. The currents entering and leaving the phantom may be resolved into $E(Y'/2 + Y'')e^{-\gamma x}/4$, leaving 1, 2 and entering 3, 4 and $EY'e^{-\gamma x}/4$, leaving 1, 2 and 3, 4 in parallel and entering Σ . The precise distribution of this last component depends upon the structure of the system, but as it flows symmetrically down the two sides of the phantom, it cannot introduce crosstalk. The first component divides equally between the two ends of the line, and this is the capacity unbalance current. The currents are therefore completely shown in Fig. 4. Allowing for the attenuation of the currents in reaching the ends of the line, we have the formulæ given below:

If a phantom (1, 2-3, 4) and its side circuit (1-2) have the transmission constants (K, Γ) and (k, γ) and the length (l) and are symmetrical throughout, then the crosstalk which will be introduced between them by the addition, at the distance (x) from the transmitting end, of small impedances (r, s, t, u) in the conductors, mutual impedances (m, n, p, q) between the conductors of the two side circuits and direct admittances ($a, b, c, d; e, f, g, h$) between the conductors of the two side circuits and between these conductors and the remainder

of the system is:

$$\Delta I = E \left(\frac{X + Z}{8kK} + \frac{Y}{16} \right) e^{-(\gamma + \Gamma)x} \quad \text{at the sending end,}$$

$$\Delta I' = E \left(\frac{X + Z}{8kK} - \frac{Y}{16} \right) e^{-[(\gamma - \Gamma)x + \Gamma l]} \quad \text{at the distant end,}$$

where $X = (r - s)$,

$$Y = 2(a - b - c + d) + (e - f),$$

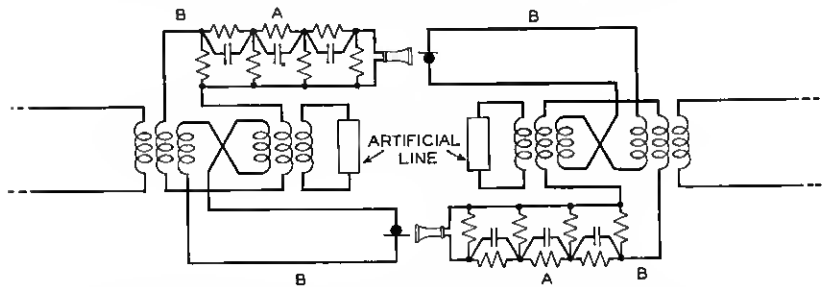
$$Z = (m - n - p + q),$$

and the positive direction in both circuits is the same as in conductor 1.

III. REPEATER CIRCUITS *

The following points seem to merit an experimental trial.

1. *A Two-way Repeater Circuit Including Two Repeaters, Each Operating as a One-way Repeater Only, as Illustrated by the Following Sketch*



With this circuit the allowable unbalance is about double that with our present standard circuit. In addition to this, singing will not be introduced by any possible unbalance, however large, in either of the lines, provided the unbalance of the other line does not exceed a certain critical magnitude. Furthermore, the two lines connected together may differ radically in character since each is balanced separately against its own artificial line.

The present standard circuit or any one of several other repeater circuits may be substituted in place of the basic circuit shown in this sketch.

Although the circuit requires that all of the repeating apparatus be duplicated and that two artificial lines (of which at least one must be a close copy of the corresponding actual line as regards telephonic

* Dated March 7, 1912.

sending end impedance) be added, it still seems to me probable that the improved performance of the circuit will prove in this extra equipment.

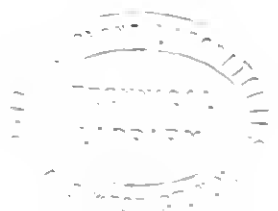
2. The Use of Repeaters of Small Amplification at Periodic Intervals along the Line

Theoretically, a given total amplification can be secured with a larger singing margin if it is distributed among a number of properly spaced points along the line rather than concentrated at a single point. For example, four equally spaced repeaters each giving an amplification of five miles might be substituted for a single repeater giving twenty miles. If the circuit suggested by the above sketch were employed this would mean a total of eight repeater elements of which four would be used, one after another, as one-way repeaters in each direction. This raises the old question as to whether equally good quality can be obtained when several repeaters are used in securing a given amplification. This point seems worth further direct experimental investigation; one step in the right direction has probably been made by raising the natural period of the diaphragm.

3. The Use of a Compensating Device Such as an Artificial Line to Reduce the Amplification at the Resonant Frequencies to the Level of the Amplification at Other Telephonic Frequencies

In the sketch, equalizing artificial lines are shown at AA; obviously the same result may be secured by introducing them at any of a number of other points in the circuit. In this way the singing margin can be increased and the quality be somewhat improved, without materially reducing the telephonic amplification. But on general principles it would seem desirable to carry the equalization as far as possible in the repeater itself. The variability of the repeater sets a limit to what may be accomplished by any compensating device which reduces the total amplification by an invariable amount at each frequency and thereby increases the percentage variation. If it became necessary merely to eliminate certain frequencies lying outside of the range required for telephony, the use of an artificial selecting circuit would seem to present no difficulty.

The variation as well as the average amplification obtained from repeaters should be investigated. When these data have been obtained for the best type of repeater it will be possible to determine whether any material benefits can be derived by the introduction of compensating circuits.



4. *Use of Two Lines for a Portion of the Route*

The circuit shown above enables us to switch two-way transmission from a single line to a pair of lines and vice versa, since any amount of line may be inserted at *BBBB*. In connection with these lines any number of additional one-way repeaters may be inserted. The operation of the system is left unchanged beyond the change in the effective amplification which is equal to the difference between the repeater amplification and the attenuation of the inserted line. In case the total attenuation of the pair of lines exceeds the total amplification of all the repeaters at every frequency the system cannot sing whatever lines be connected at the ends. Practical applications will hinge upon the possibility of securing good quality from a number of repeaters used in sequence.

Suggestions 1 and 2 seem sufficiently promising to warrant some experimental work at an early date.

I am preparing a discussion of the general repeater circuit including any number of repeating elements and shall present the theoretical deductions applicable to the above suggestions in that memorandum.

